Spectral Methods in Time Series Analysis

Using the SSA-MTM Toolkit for Time Series Analysis

- *Fereshteh*
- *Silvia*
- *Rosana*

Singular Spectrum Analysis

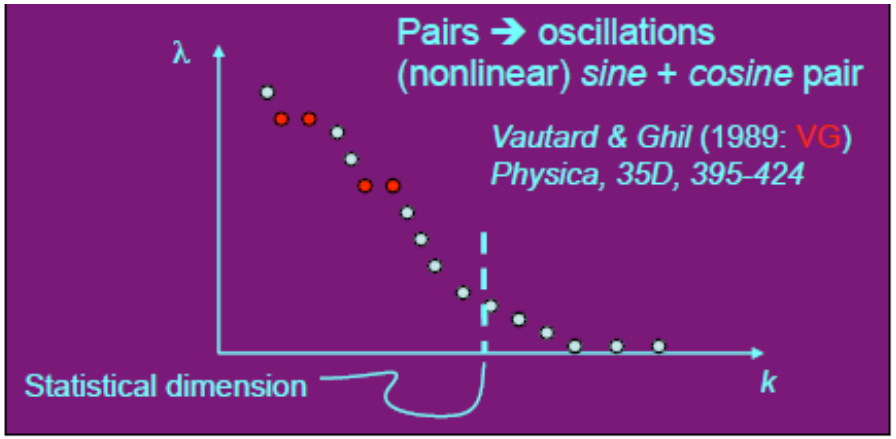
- Allows to get spectral information on the time series

  time series X(t): $t = 1,.....,N$

- X(t) embedded in a vector space of dimension M:
  - Diagonalization of the MxM lag-covariance matrix C_X
  - Calculation of the M eigenvectors E_k of the lag-covariance matrix C_X ; E_k are called temporal empirical orthogonal functions (EOFs).

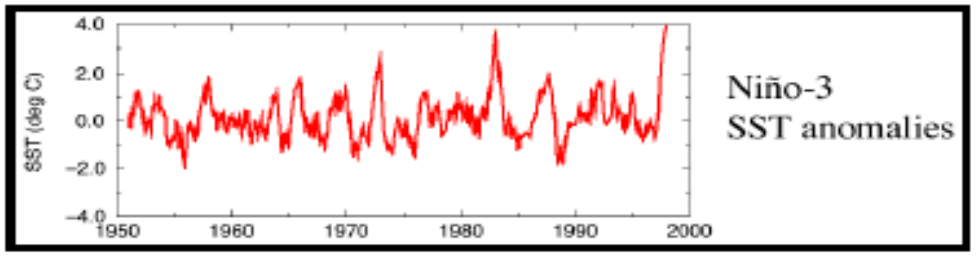
Decomposition and Reconstruction

- Projecting the time series onto each EOF yields the corresponding temporal principal components (PCs).
- The eigenvalues λ_k of C_x account for the partial variance in the direction E_k.
- A pair of nearly equal λ_k and associated EOFs in approximate phase quadrature characterize an oscillatory mode.

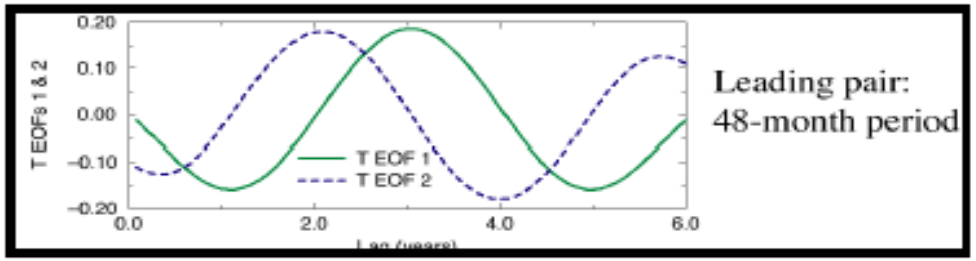


From M_Ghil-SSA-BESS Presentation

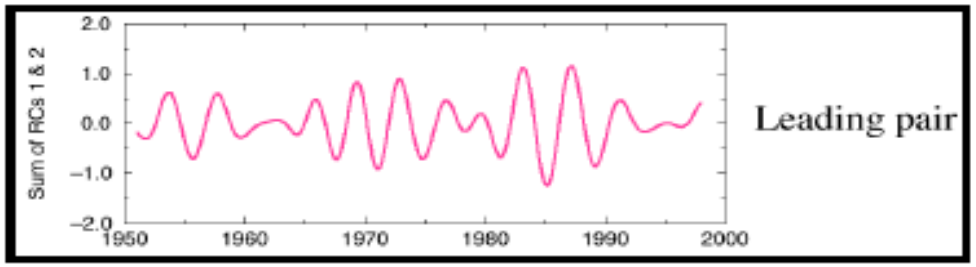
Time series



T-EOFs



RCs



From M_Ghil-SSA-BESS Presentation

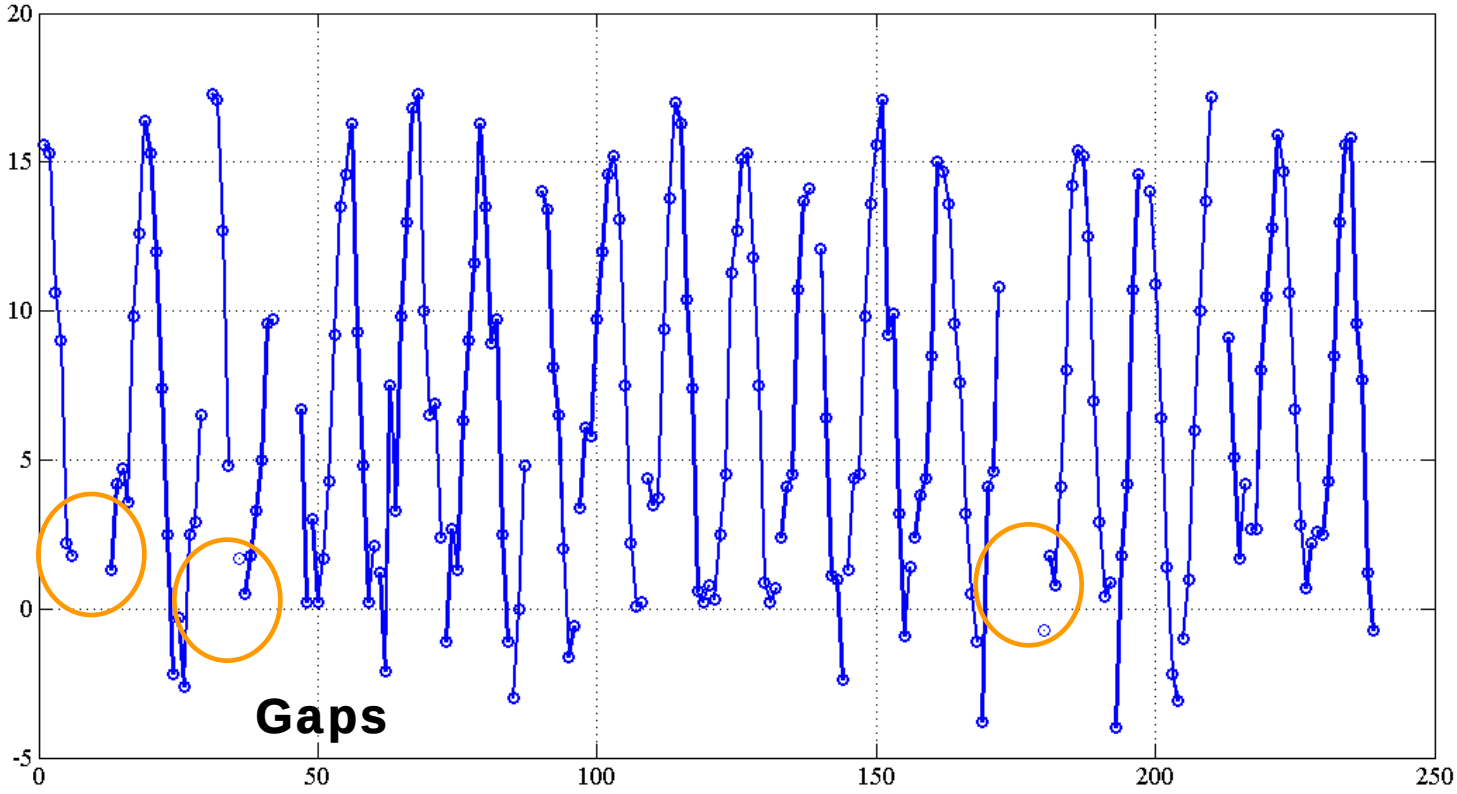
Window Length (M) and Principal Component Number (PC)

- Larger window size improves resolution but at the cost of increasing variance of the spectral estimates.
- The minimum number of PCs that properly reproduces the signal should be used.

SSA-MTM Toolkit for TS Analysis

http://www.atmos.ucla.edu/tcd/ssa/

- The Singular Spectrum Analysis - MultiTaper Method Toolkit is a software program to analyze short, noisy time series.



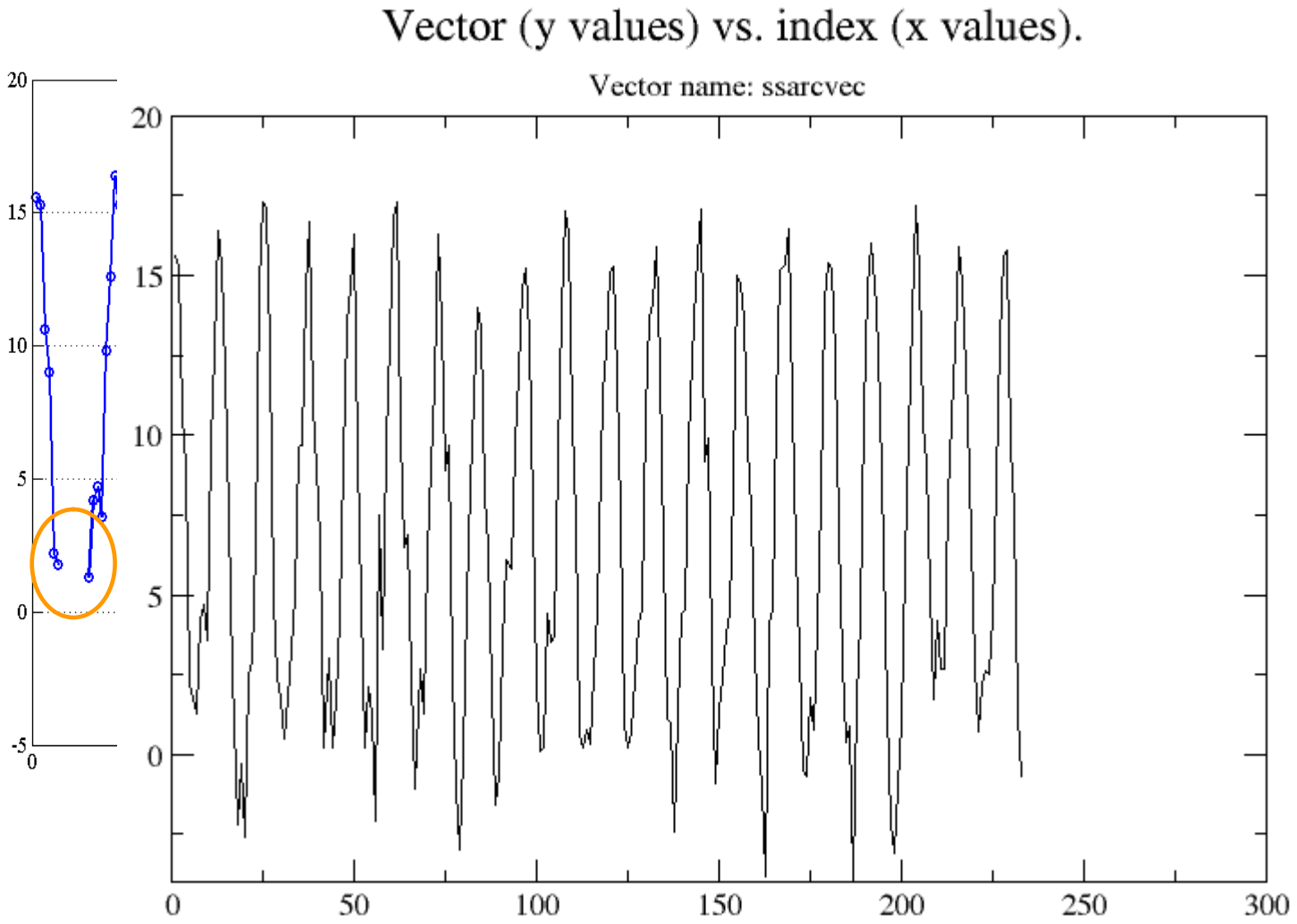
Monthly Mean Temperature in Acceglio, NW Italy (1989-2008)

Gap filling

- Decomposition of the time series into PCs.
- Reconstruction of the signal using first significant components and so the missing values are estimated.

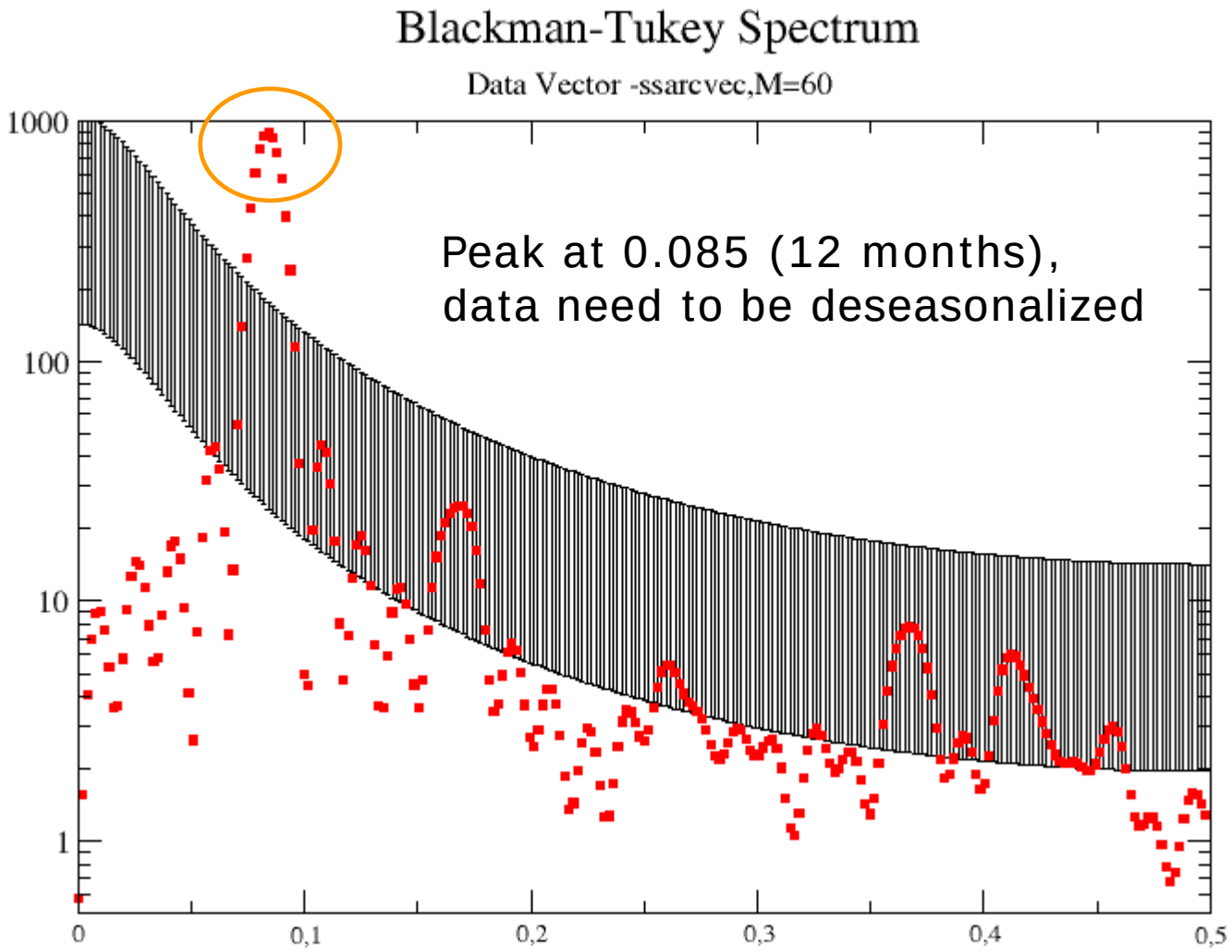
Temperature Time Series

Reconstruction and Gap-Filling (M:60, PC:12)



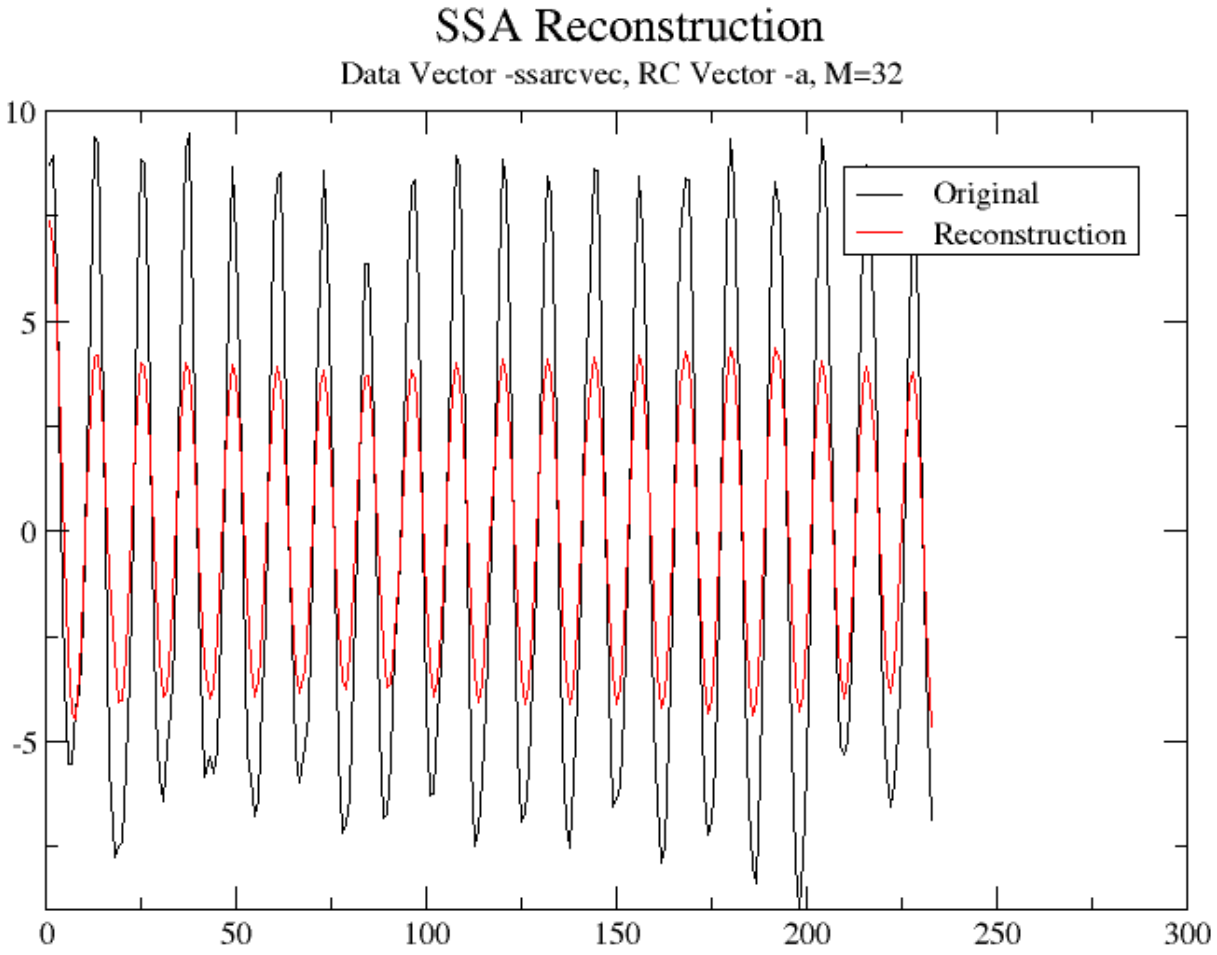
Temperature Time Series

B-T Correlogram – Reconstructed TS



Temperature Time Series

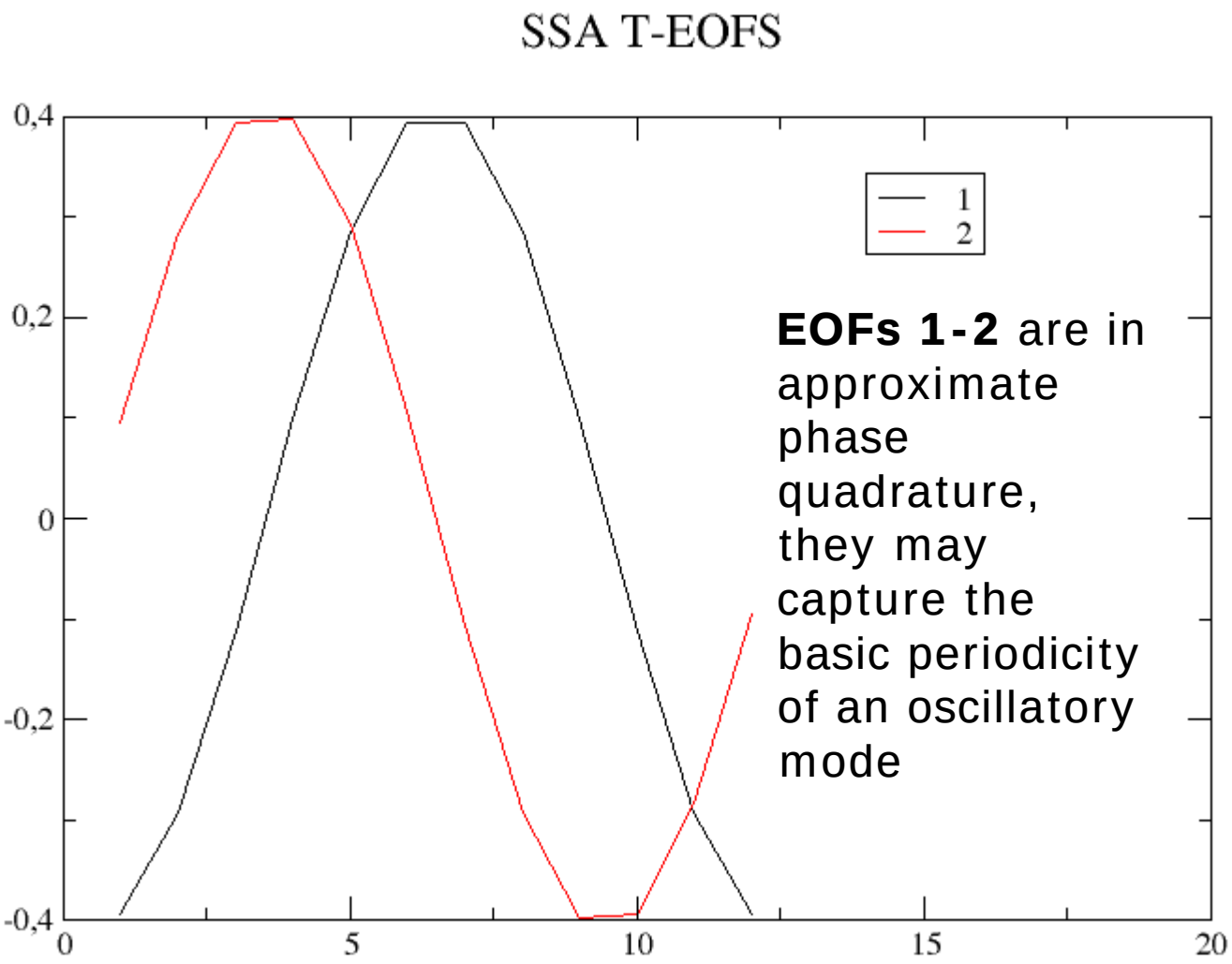
Principal Component (PC) = 1



Analysis of **PC 1** which captures the seasonal cycle of the time series

Temperature Time Series

EOFs 1-2



EOFs 1-2 are in approximate phase quadrature, they may capture the basic periodicity of an oscillatory mode

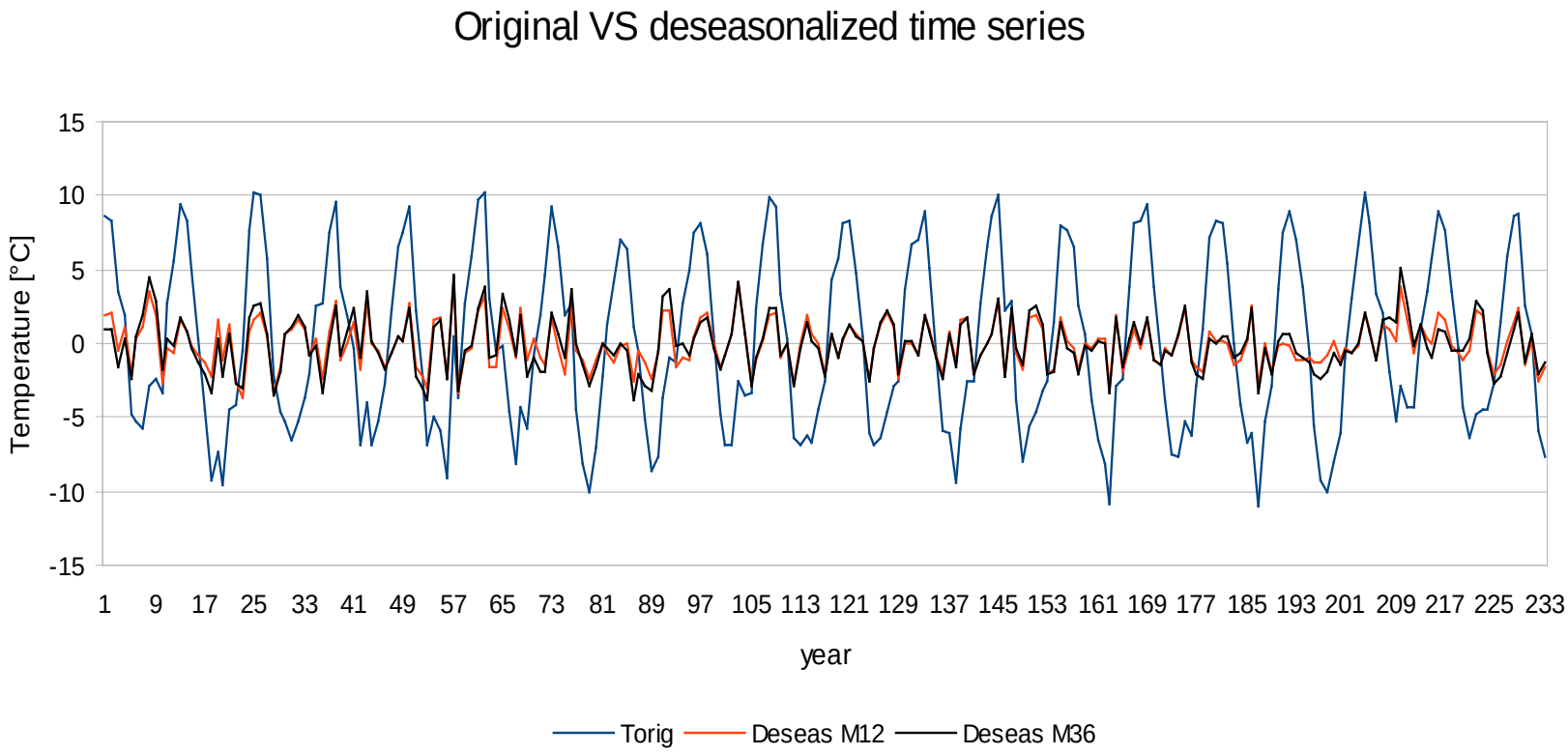
Deseasonalization of Time Series

Seasonal components of the TS need to be removed, therefore PC 1-2 are taken away from the original TS.

- Deconstruction of the time series in PCs using a window length of 12 months (M=12).
- Reconstruction of the seasonal cycle using the PC1-2.
- Substraction of PC1-2 from the original time series.

Temperature Time Series

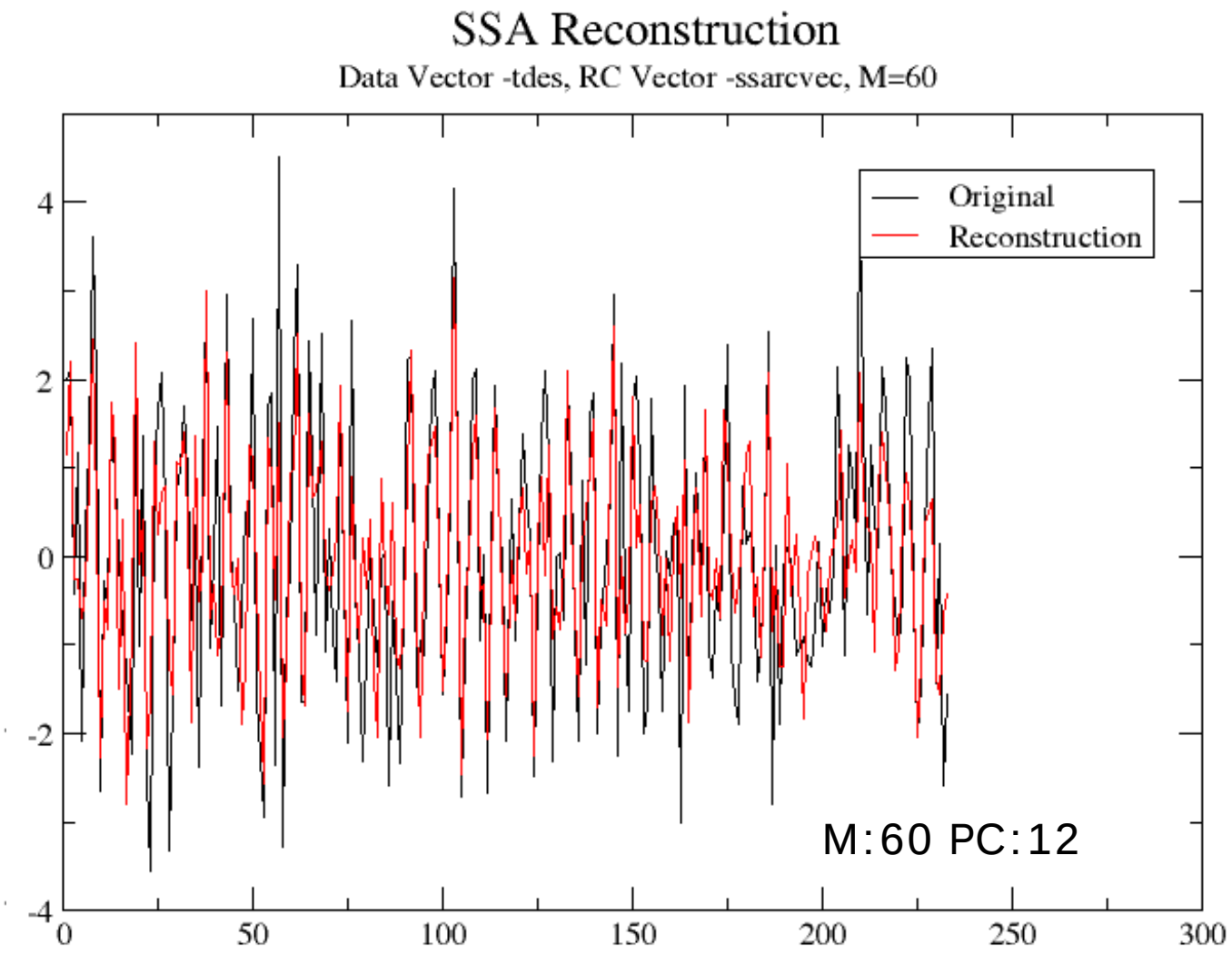
Deseasonalization



M=12 and M=36 produce similar deseasonalized time series

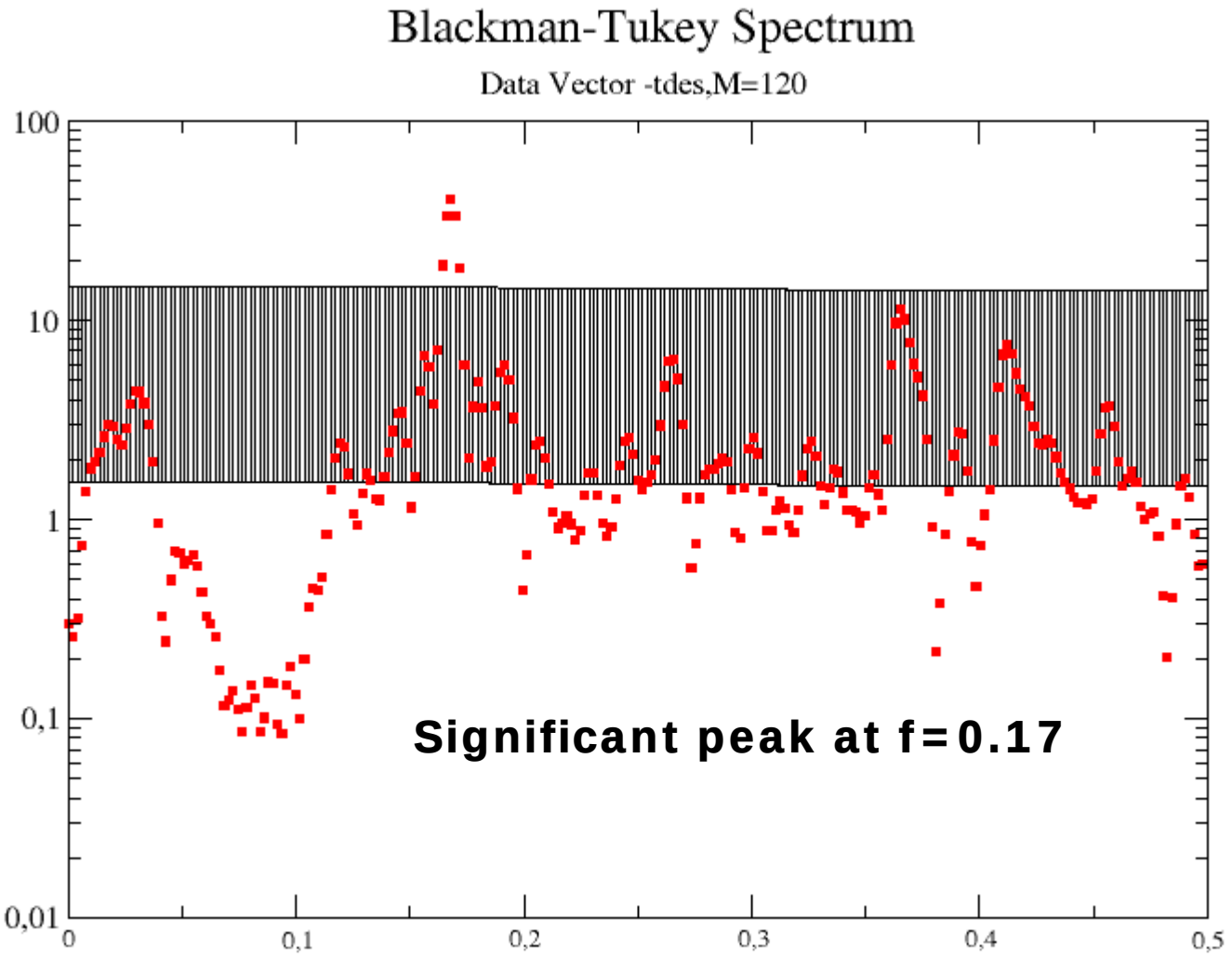
Deseasonalized Temperature TS

Reconstruction



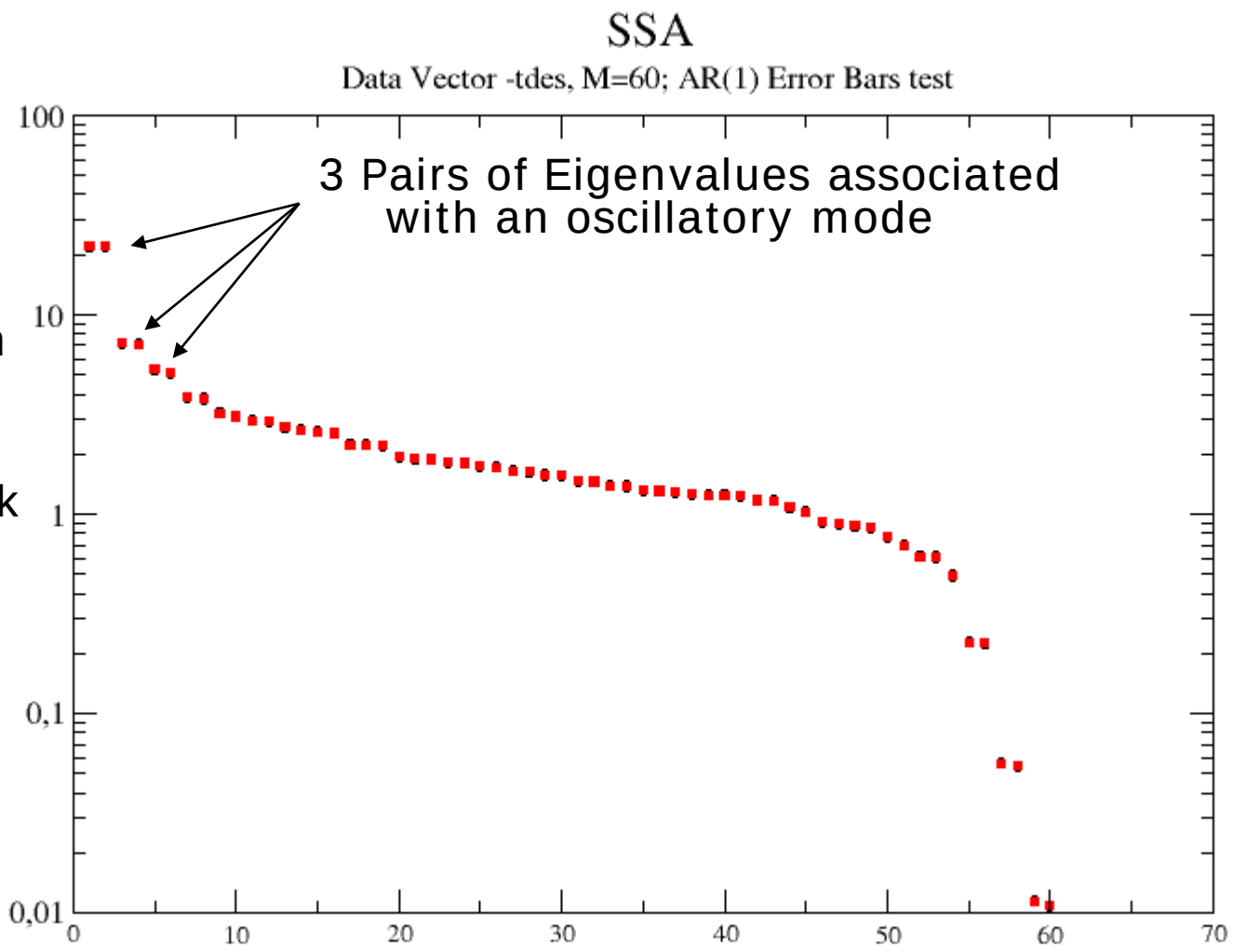
Temperature Time Series

B-T Correlogram – Deseasonalized TS



Deseasonalized Temperature TS Scree Diagram

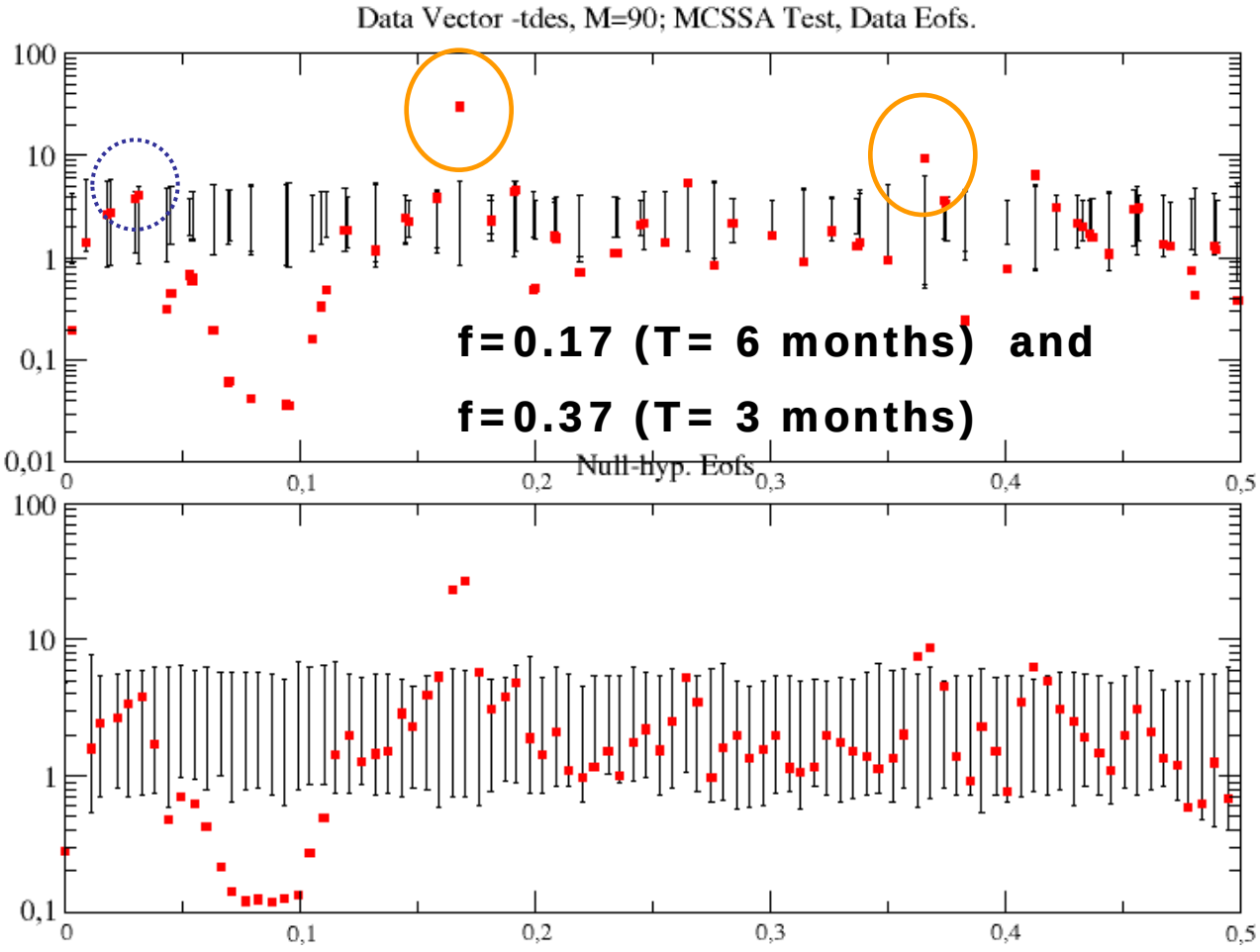
Eigenspectrum displayed in order of eigenvalue rank



Deseasonalized Temperature TS

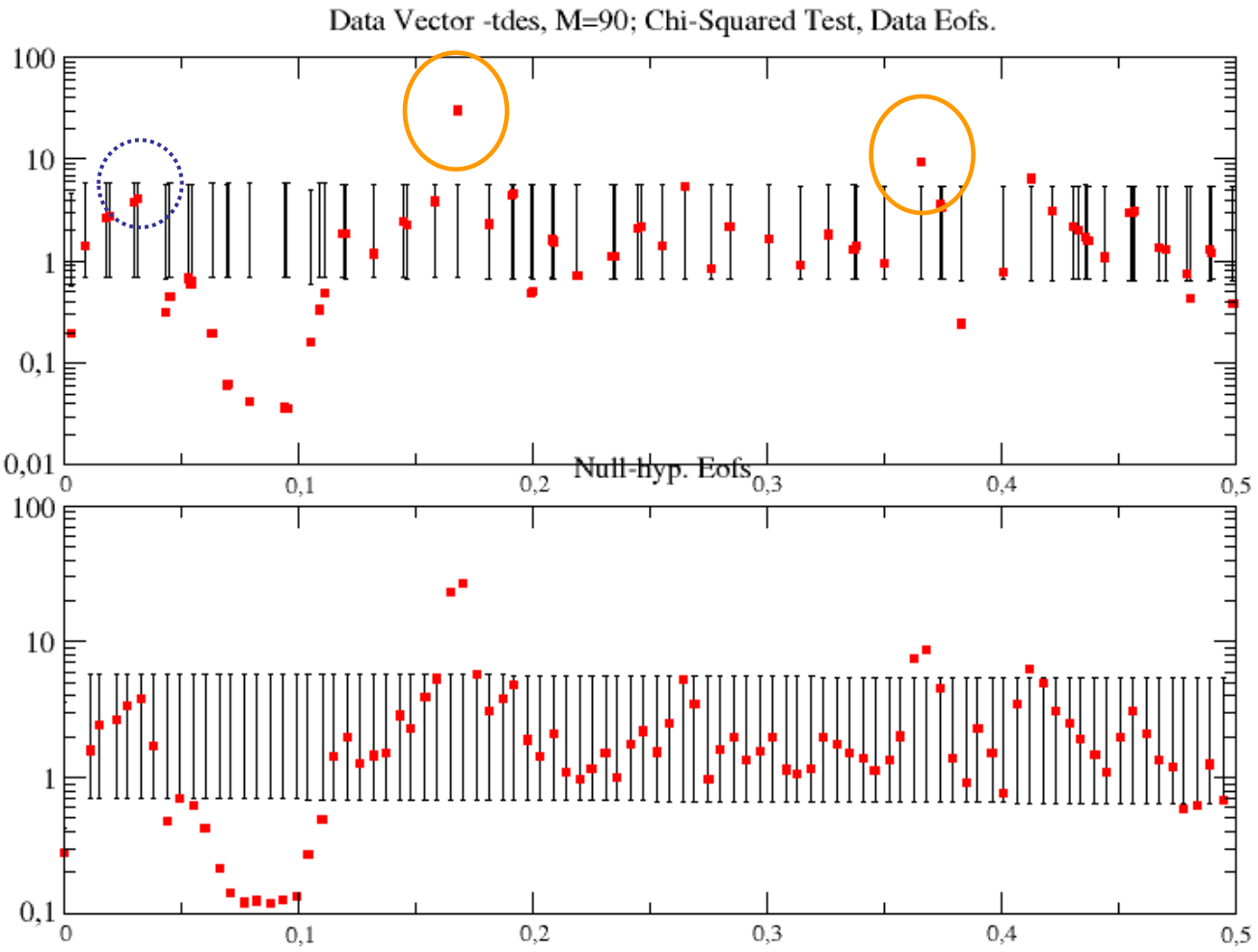
MCSSA Significance Test

Tests significance against red noise null hypothesis



Deseasonalized Temperature TS

Chi-Squared Significance Test



Conclusions

- The Toolkit has provided a means to fill gaps and remove the seasonal cycle of the temperature TS.
- However, in order to obtain spectral peaks at lower frequencies this temperature TS should be much longer.

Thank you for your attention!